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LIQUID METAL HEAT TRANSFER IN TURBULENT PIPE FLOW WITH UNIFORM WALL FLUX

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Abstract — This paper presents an investigation of the thermal entry-region problem for liquid metal turbulent flow inside a round pipe. The effect of axial conduction is taken into account in both upstream and downstream directions. The exact solution is obtained by using the method of separation of variables for parameters, $5 \leqslant Pe \leqslant 1000$ and $0.001 \leqslant Pr \leqslant 0.02$ with $Re > 4000$. The Prandtl numbers cover the full range of liquid metals.

Excellent agreement is observed upon comparison of the resulting Nusselt numbers with the available experimental data in both the entrance and fully developed regions. The solution also indicates that the fully developed Nusselt number depends only on the Reynolds number at low Peclet numbers. The interpolation formula

$$Nu_\infty = 3.01\ Re^{0.0833}$$

is found to fit the calculated data well for $Pe < 100$.

NOMENCLATURE

D, R_0,	diameter and radius of the pipe;
DF,	damping factor, equation (8);
f, g,	quantities defined in equation (2);
f_s,	friction factor;
h,	heat transfer coefficient;
k,	thermal conductivity;
l, l^+, l_0^+,	dimensional, dimensionless damped and undamped mixing lengths, $l^+ = (DF)l_0^+$, $l_0^+ = V^*l/\nu$;
Nu,	Nusselt number, hD/k;
Pe,	Peclet number, VD/α;
Pr, Pr_t,	Prandtl and turbulent Prandtl numbers, $Pr = \nu/\alpha$, $Pr_t = \varepsilon_m/\varepsilon_h$;
q,	radial heat flow rate;
R, X,	cylindrical coordinates;
r, x,	dimensionless cylindrical coordinates, $r = R/R_0$, $x = 4X/Pe\ D$;
Re, Re^*,	Reynolds number and shear Reynolds number, $Re = VD/\nu$, $Re^* = V^*D/\nu$;
T, T_0,	temperature and temperature at $X = -\infty$;
U, u,	dimensional and dimensionless velocity, $u = U/V = u^+/u_m^+$;
u^+,	dimensionless velocity, U/V^*;
V,	mean velocity;
V^*,	shear velocity, $V(f_s/8)^{1/2}$;
Y, y, y^+,	defined as $Y = R_0 - R$, $y = Y/R_0$ and $y^+ = V^*Y/\nu$.

Greek symbols

α,	thermal diffusivity;
ε, ε_x,	dimensionless total eddy diffusivity in r and x directions respectively, $\varepsilon = 1 + \varepsilon_h/\alpha$, $\varepsilon_x = 1 + (\varepsilon_h)_x/\alpha$;
ε_m, ε^+,	dimensional and dimensionless radial eddy viscosity, $\varepsilon^+ = \varepsilon_m/\nu$;
ε_h, $(\varepsilon_h)_x$,	radial and axial eddy diffusivity;
θ,	dimensionless temperature, $(T - T_0)/(q_w D/2k)$;
ν,	kinematic viscosity;
σ,	ratio of axial to radial total eddy diffusivity, $\varepsilon_x/\varepsilon$.

Subscripts

b,	bulk;
c,	center of pipe;
m,	mean;
w,	wall;
∞,	fully developed region, $x = \infty$.

INTRODUCTION

WIDESPREAD interest has appeared in the use of liquid metals as heat transfer media because of their high boiling points and resistance to thermal decomposition. Qualitatively, it is also recognized that the heat transfer coefficients are higher in liquid metals than any other fluid for a given system and pumping power. Therefore, liquid metals have been widely used as a coolant in nuclear reactors.

In spite of the attractions and the potentialities of the uses of liquid metals in industrial applications, the mechanism of turbulent heat transfer in liquid metals is not well understood. In the early literature [1–3] the turbulent Prandtl number, Pr_t, which is the ratio of eddy viscosity to eddy diffusivity, was taken to be a constant or unity. Various experiments [4, 5] were conducted to determine the turbulent Prandtl number from measured temperature and velocity profiles. It was generally agreed that Pr_t is larger than unity for liquid metals and is not a constant across the pipe. Later, Azer and Chao [6] proposed a mechanism of turbulent heat transfer based on a modification of

Prandtl's mixing length hypothesis which assumes that there is a continuous change of momentum and energy during the movement of the eddy. Two expressions giving the turbulent Prandtl number were obtained. One was for liquid metals and the other for fluids of the Prandtl number ranging from 0.6 to 15. Computations of the Nusselt number and the temperature profile in liquid metals were carried out under the wall condition of constant heat flux using the deduced expression for turbulent Prandtl number. Comparison of Nusselt number with the prediction of Lyon [2] was made. The improvement over Lyon's result is obvious. According to their results, the lowest possible Nusselt number is 7, but the bulk of experimental data [7] at low Peclet numbers indicate a value considerably less than 7. Azer and Chao [8] also used the theory to investigate liquid metal heat transfer in fully developed pipe flow with constant wall temperature. Recently, Notter and Sleicher [9] solved the turbulent Graetz problem numerically. A new empirical turbulent Prandtl number expression was proposed and recommended for use together with values of eddy viscosity described in literature [10] to predict values of eddy diffusivity for $Pr < 1$. Özişik *et al.* [11–13] applied this model to the freezing of liquids of low Prandtl number in turbulent flows. Another model was proposed by Na and Habib [14] based on a modified form of the mixing length theory developed by Cebeci [15] for turbulent pipe flows. The model predicts the fully developed Nusselt number with $Pr = 0.02$–14.3 under uniform wall flux condition and fits the available experimental data well for a range of the Peclet numbers. Both the eddy viscosity and eddy diffusivity were given in terms of the Nikuradse's [16] mixing length and damping factors, and the turbulent Prandtl number was given by the ratio of the two. It is interesting to note that the bulk of experimentally determined Nusselt numbers are lower than those predicted by the existing theoretical analyses, especially in the low Peclet number range [6–8].

Although axial conduction can be ignored in turbulent convection for nonmetallic fluids, this might not always be justified for liquid metals. In fact, the Peclet number can be as small as 5 in turbulent pipe flow for liquid metals. A number of prior works have investigated the effect of axial conduction on laminar heat transfer [17–20], but few have considered this effect in the turbulent flow case. In turbulent flow, the effect was first considered by Schneider [21]. Schneider's investigation was based on observing that with small Peclet numbers, say less than 100, axial conduction is significant. He considered the case of a liquid metal flowing through a round pipe with uniform wall temperature. The velocity was assumed to be uniform throughout the pipe. The inlet temperature was taken to be uniform (at $X = 0$) and the pipe is sufficiently long so that axial conduction is negligible before the pipe outlet is reached. Both the dimensionless axial and radial eddy diffusivity were assumed to be equal to unity ($\varepsilon_x = \varepsilon = 1$). Based on the assumptions, he concluded that for the purpose of computing the overall heat transfer rate, axial conduction can be neglected when the Peclet number is larger than 100. Recently, Lee [22] also investigated the effect of axial conduction on a problem of liquid metal heat transfer in turbulent pipe flow with uniform wall temperature. To allow for the effect of upstream conduction, the fluid temperature was taken to be uniform at $X = -\infty$ and the pipe wall was adiabatic at $X \leq 0$. The results led to the conclusion that the effect is significant in the thermal entry-region for $Pe < 100$, but is negligible in the thermally fully developed region. Ignoring the axial conduction was shown to result in an error of about 40% (low) in Nusselt number at a low Peclet number, say $Pe = 5$.

It is noted that previous publications have not given predictions or data at low Peclet numbers ($Pe < 100$) and low Reynolds numbers ($Re < 10^4$) for turbulent heat transfer. Under such conditions, the axial conduction effect may be significant in the entrance region and the influence of boundary sublayers must be taken into account. The purpose of this paper is to investigate the turbulent heat transfer in the thermal entry-region, considering the effect of axial conduction. The Peclet numbers considered are less than 10^3. A modified turbulent model taking into account the damping effect of the wall on the mixing length is adopted in this work to approximate the eddy viscosity. The laminar sublayer and buffer zone are also considered in the flow field. Comparisons between the theoretical Nusselt numbers and the existing experimental data are made in both the entrance and fully developed regions.

THEORETICAL ANALYSIS

Consider a liquid metal flowing through a round pipe of infinite length. The velocity is turbulent and fully developed, and the fluid temperature is taken to be uniform at $X = -\infty$. The pipe wall is adiabatic at $X \leq 0$ but heated with a constant wall flux at $X \geq 0$. The flow is steady, Newtonian and incompressible. The physical properties are constant. Viscosity dissipation, free convection and tube wall thermal resistance are all assumed to be negligible.

After imposing the assumptions and introducing the dimensionless transformations, $U = uV$, $R = rR_0$, $Pe = VD/\alpha$, $X = x(Pe\,D/4)$, $\varepsilon_x = 1+(\varepsilon_h)_x/\alpha$, $\varepsilon = 1+\varepsilon_h/\alpha$, $\sigma = \varepsilon_x/\varepsilon$, $\theta = (T-T_0)/(q_w D/2k)$, the energy equation becomes

$$f\,\partial\theta/\partial x = g\,\partial\theta/\partial r + \partial^2\theta/\partial r^2 + (4\sigma/Pe^2)\partial^2\theta/\partial x^2 \quad (1)$$

in which

$$f = u/\varepsilon \quad \text{and} \quad g = 1/r + (\mathrm{d}\varepsilon/\mathrm{d}r)/\varepsilon \quad (2)$$

and the associated boundary conditions are

$$\theta(-\infty, r) = 0, \quad \theta(\infty, r) = \theta_{fd}, \quad \partial\theta(x,0)/\partial r = 0,$$

$$\partial\theta(x,1)/\partial r = 0 \quad \text{for } x \leq 0,$$

$$\partial\theta(x,1)/\partial r = 1 \quad \text{for } x \geq 0 \quad (3)$$

where θ_{fd} refers to the dimensionless fully developed temperature profile which includes the effect of axial conduction.

As in Schneider [21] and Lee [22], the axial eddy diffusivity is assumed to be equal to the radial eddy diffusivity, that is, $\varepsilon_x = \varepsilon$ or $\sigma = 1$. It is noted that a fully developed flow idealization implies that both dimensionless velocity profile u, and total eddy diffusivity ε, are independent of x. The LHS of the energy equation (1) is the forced convection term, the first two terms on the RHS are the radial conduction term and the last term, $(4\sigma/Pe^2)(\partial^2\theta/\partial x^2)$, usually neglected for large Peclet number flows but rather important for low Peclet number flows, is the so-called axial conduction term. For liquid metals ($0.001 \leqslant Pr \leqslant 0.02$), the Peclet number can be as small as 5 in turbulent pipe flow. Therefore, this term is considered in the work. To solve the energy equations (1) and (2) with the associated boundary conditions (3), the functions u, ε and θ_{fd} must be determined.

In this work, the von Kármán's three region logarithmic velocity profile and the friction law are adopted. They are

$$\begin{aligned} u^+ &= y^+ && \text{if } 0 \leqslant y^+ \leqslant 5,\\ u^+ &= 5.0 \ln y^+ - 3.05 && \text{if } 5 \leqslant y^+ \leqslant 30, \\ u^+ &= 2.5 \ln y^+ + 5.5 && \text{if } y^+ \geqslant 30, \end{aligned} \tag{4}$$

and

$$1/f_s^{1/2} = 2.0 \log (Re\, f_s^{1/2}) - 0.8 \tag{5}$$

where

$$y^+ = V^*(R_0 - R)/v \quad \text{and} \quad u^+ = U/V^*.$$

The dimensionless velocity u then can be taken as

$$u = u^+/u_m^+ \tag{6}$$

where u_m^+ is the mean value of the dimensionless velocity u^+ over the cross-sectional area of the flow through the tube.

The eddy viscosity

$$\varepsilon^+ = (l^+)^2(\mathrm{d}u^+/\mathrm{d}y^+) \tag{7}$$

where

$$\begin{aligned} l^+ &= l_0^+(DF),\\ l_0^+ &= V^*l/v = (V^*R_0/v)(l/R_0) = (Re^*/2)(l/R_0)\\ &= (Re^*/2)(0.4y - 0.44y^2 + 0.24y^3 - 0.06y^4),\\ DF &= 1 - \exp(-y^+/26) \end{aligned} \tag{8}$$

and the Azer–Chao turbulent Prandtl number [6]

$$Pr_t = \frac{1 + 380\, Pe^{-0.58} \exp(-y^{0.25})}{1 + 135\, Re^{-0.45} \exp(-y^{0.25})} \tag{9}$$

are used to approximate the eddy diffusivity

$$\varepsilon = 1 + Pr\, \varepsilon^+/Pr_t. \tag{10}$$

The expression of the turbulent Prandtl number was formulated from Prandtl's mixing length hypothesis but modified for a continuous change of momentum and energy during the movement of the eddy. The expression is available for liquid metals only. Note that the present model is different from that of Azer and Chao [6] because of the use of the damping factor DF. It is widely held that the conventional mixing length l_0^+ overstates the effect of turbulence in the immediate neighborhood of the wall. The damping factor DF proposed by van Driest [23] is adopted to account for the damping effect of the wall. The laminar sublayer and buffer zone ignored in the literature [6] are also considered in the velocity profile. The influence of the boundary sublayers on the Nusselt number may be significant at $Re < 10^4$.

In the thermally fully developed region, one can obtain the solution by solving the energy equation without the axial conduction term and the solution is

$$\theta_{fd} = 2x + 2\int_0^r (1/r\varepsilon)\int_0^r ru(\mathrm{d}r)^2 + C \tag{11}$$

where the integration constant C can be determined from a heat balance taken over the region extending from $x = -\infty$ to an arbitrary axial position in the thermally fully developed region and the result is

$$C = (8/Pe^2)\int_0^1 2\varepsilon\sigma r\,\mathrm{d}r - 4\int_0^1 ru \times \int_0^r (1/r\varepsilon)\int_0^r ru(\mathrm{d}r)^3. \tag{12}$$

It is noted that the fully developed temperature profile θ_{fd} differs from that when axial conduction is neglected by the additional term

$$(8/Pe^2)\int_0^1 2\varepsilon\sigma r\,\mathrm{d}r.$$

For laminar pipe flow, $u = 2(1-r^2)$, $\varepsilon = 1$ and $\sigma = 1$, the fully developed temperature profile is

$$\theta_{fd} = 2x + r^2 - r^4/4 - 7/24 + 8/Pe^2. \tag{13}$$

After finding u, ε and θ_{fd}, the functions $f(r)$ and $g(r)$ can be obtained from equation (2) and the energy equation (1) may be solved immediately by the method of separation of variables. For each pair of Pe and Pr, the first 20 eigenvalues and the corresponding eigenfunctions are determined by using a trial and error technique and the Runge–Kutta method in both the regions $x \leqslant 0$ and $x \geqslant 0$. Through the use of the Gram–Schmidt orthonormalization procedure, the two series expansion coefficients are determined from the boundary conditions at $x = 0$. Both the temperature distribution and the axial temperature gradient must be equally matched on both sides of $x = 0$. A detailed procedure has been described [17]. After determining the series coefficients the temperature distribution can be obtained in both regions $x \leqslant 0$ and $x \geqslant 0$, and the local bulk temperature is evaluated by

$$\theta_b(x) = 2\int_0^1 ru\theta(x,r)\,\mathrm{d}r. \tag{14}$$

The local Nusselt number $Nu(x)$ can be written as

$$Nu(x) = hD/k = 2/(\theta_w - \theta_b). \tag{15}$$

It is noted that when the Peclet number is large, the present method is still available but the eigenvalues scatter in a random pattern and the calculations are very time consuming. Fortunately, a method used [19] for the case of no axial conduction is applicable in present work for $Pe > 1000$. Under this condition, the effect of axial conduction is negligible.

RESULTS AND DISCUSSION

Three models of turbulent heat transfer have been proposed by Azer and Chao [6], Notter and Sleicher [9], Na and Habib [14]. In this investigation, a modified form of the Azer–Chao model is adopted. The parameters taken in this work are $Pe = 5, 10, 20, 40, 60, 80, 100, 300, 500, 700, 1000$ and $Pr = 0.001, 0.004, 0.01, 0.02$ with $Re > 4000$.

Figure 1 illustrates the temperature distribution along the axial position for the case of $Pe = 5$ and $Pr = 0.001$. The temperature profile is lower and more uniform at entrance, then increases along the longitudinal direction and finally develops into the fully developed profile θ_{fd}. A subsidiary figure for $\theta - \theta_c$ is also provided in the figure where the subscript c stands for the center of the pipe. The series expansion coefficients in both regions $x \leqslant 0$ and $x \geqslant 0$ satisfy the matching conditions required at $x = 0$. As can be observed, the temperature distributions at $x = -0$ and $x = +0$ are equally matched (except near the wall). This is, in fact, a direct proof of the mathematical correctness of the present solution. It is noted that at the pipe wall, the radial temperature gradient is zero at $x < 0$ and unity at $x > 0$. This discrepancy in boundary conditions causes a temperature discontinuity near the pipe wall at position $x = \pm 0$. The temperature profiles are seen to be strongly affected by the effect of upstream conduction. For example, at $x = +0$, the value increases from 0.2312 at the center to 0.4899 at the wall. If the effect of axial conduction is assumed to be negligible, the temperature at $x = -0$ then is zero [9, 10]. This assumption should result in a significant error in the Nusselt number.

Figures 2–5 show the local Nusselt number vs dimensionless axial coordinate x for the case of $Pr = 0.001, 0.004, 0.01$ and 0.02 respectively with the Peclet number as a parameter. It is seen that for each pair of Pr and Pe, the value of Nusselt number approaches asymptotically to a certain constant as x approaches infinity. This value depends on the profile of θ_{fd}. For a given Prandtl number, the flow with larger Peclet number yields greater fully developed Nusselt number. In the thermal entrance-region, increasing the Peclet number should decrease the effect of axial conduction and the temperature profile tends to be more uniform at $x = 0$. This results in a higher Nusselt number as x approaches zero. Hence, for a given Prandtl number the Nusselt curves do not cross each other as shown in each of the four figures. This is different from that of laminar heat transfer [17]. Note that the dimensionless axial coordinate x which is defined in terms of Peclet number is different from the real axial position X/D. For example, the position $X/D = 4.6$ shown in Fig. 5 is not a vertical line but rather a curve. It is also noted that the contribution of axial conduction is rather important in the thermal entrance-region when $Pe < 100$. Detailed information can be referred to the study by Lee [22] in which the pipe wall at $x \geqslant 0$ was kept at a uniform wall temperature.

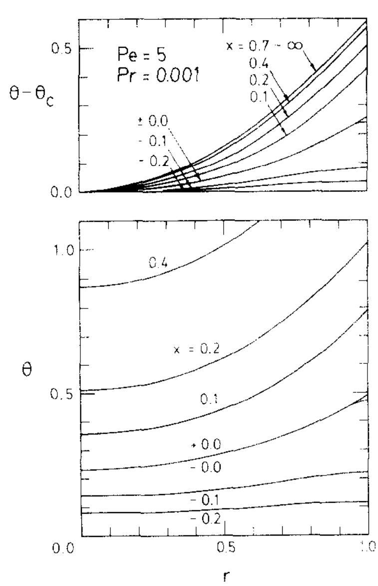

FIG. 1. Temperature variations along the flow direction for the case of $Pe = 5$ and $Pr = 0.001$.

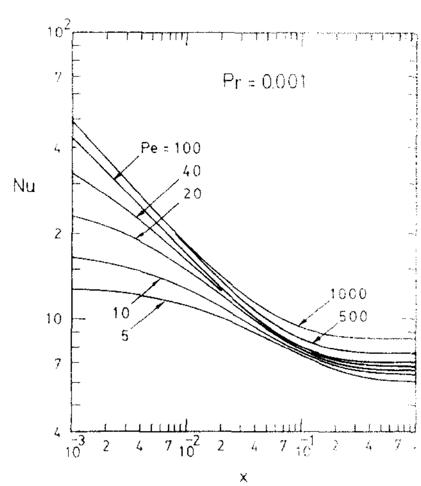

FIG. 2. Nusselt number curves for the case of $Pr = 0.001$.

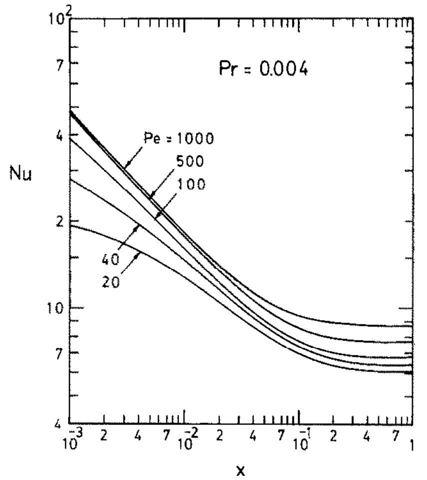

FIG. 3. Nusselt number curves for the case of $Pr = 0.004$.

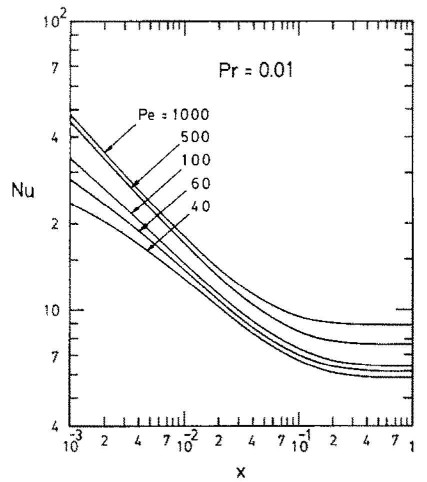

FIG. 4. Nusselt number curves for the case of $Pr = 0.01$.

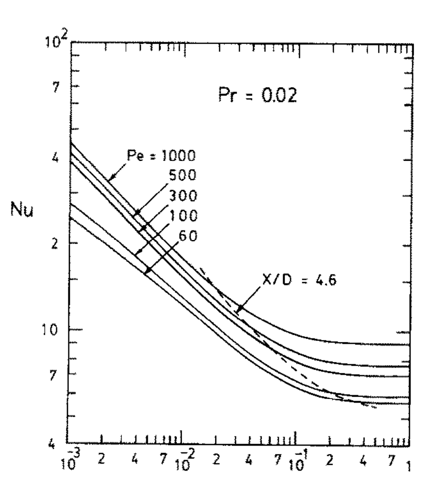

FIG. 5. Nusselt number curves for the case of $Pr = 0.02$.

To examine the reliability of the present prediction, the computed Nusselt numbers vs Peclet number correlation is compared with the existing theoretical predictions and experimental data. Experimental data for mercury and lead–bismuth have been given [24, 25]. The fluid bulk temperatures were measured at the inlet and outlet of the test section and the wall temperature was measured at eight stations along the tube. Four of the stations were located at $X/D = 4.6$, 13.8, 23 and a position near the end of the tube whose length to diameter ratio is 73.6. Figure 6 shows the comparison of Johnson's data [24, 25] with the present prediction for various Peclet numbers at $X = 4.6D$. The Prandtl number in both experiments was about 0.02. It is apparent from this figure that the present calculated Nusselt number is in excellent agreement with the experimental data. The agreement is also found to be excellent if comparisons are made at other local stations. Because of the lack of other theoretical works on the local Nusselt number in entrance region, the early predictions of Poppendiek–Palmer and Deissler [7] are plotted for comparison. The prediction of Poppendiek and Palmer was based on the assumption that the eddy diffusivity is negligible when compared with molecular diffusivity; thus this solution is independent of Prandtl number and its value is seen to be too low. Deissler obtained the solution numerically. To approximate the eddy diffusivity, he proposed a model based on a modification of Reynolds analogy by allowing for heat transferred by conduction to or from a turbulent particle as it moves radially in the tube. This result is also low. Note that the Peclet numbers presented in both works were larger than 100; thus the contribution of axial conduction is insignificant.

Comparisons of Notter–Sleicher's prediction [9] and Johnson's data [24, 25] with the present results are shown in Fig. 7 at various dimensionless axial coordinate x for the case of $Pr = 0.02$ and $Pe = 1000$. An excellent agreement is observed between the experimental data and the present results. The

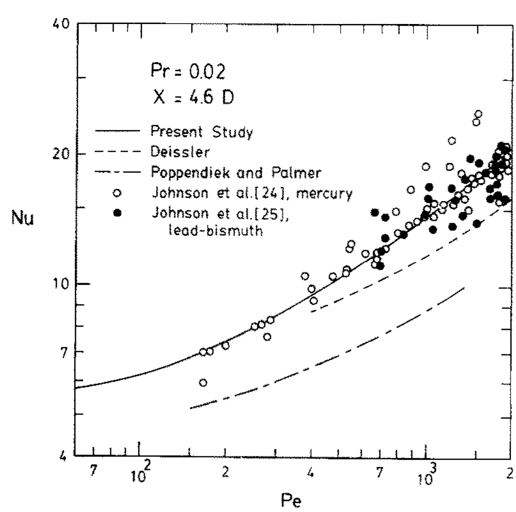

FIG. 6. Comparisons of existing experimental data and theoretical predictions with present study at $X = 4.6D$.

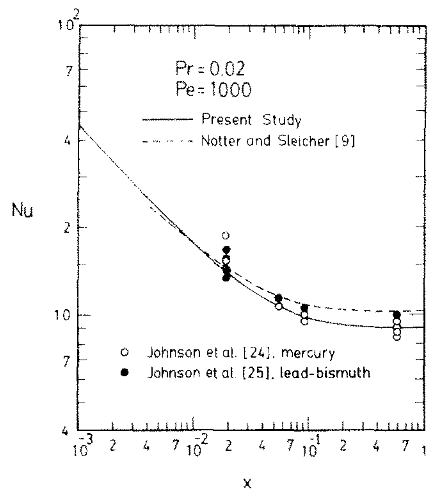

FIG. 7. Comparisons of existing experimental data and theoretical prediction with present study in thermal entrance region for the case of $Pr = 0.02$ with $Pe = 1000$.

prediction of Notter and Sleicher [9] is in good agreement with the present prediction in the entrance region, but is about 14% higher in the thermally fully developed region. Since the axial conduction is negligible throughout the pipe at $Pe = 1000$, the difference is due to the different model used.

Figure 8 shows the fully developed Nusselt number vs Peclet number for $Pr = 0.02$. Predictions and experimental data by other studies are also indicated for comparison. The solution of the present study includes the case of $Pe = 2000$. Agreement between the present prediction and the experimental data is excellent. Comparison is also made with the following prediction made by Azer and Chao [6]

$$Nu_{\infty} = 7+0.05Pr^{0.25}Pe^{0.77}. \qquad (16)$$

The improvement of the present prediction over equation (16) is obvious. In the thermally fully developed region, the axial conduction is negligible at all Peclet numbers. Therefore, this difference is due to the absence of damping effect of wall on the mixing length in their work. In addition, to obtain a simplified closed form of fully developed Nusselt number, Azer and Chao [6] assumed the dimensionless radial heat flow rate, q/q_w, to be $r^{1.75}$ which is independent of both the Reynolds number and the Prandtl number. These errors should be more pronounced at low Reynolds number, say $Re < 10^4$. Note that according to the present result, a fully developed Nusselt number less than 7 at low Peclet numbers is possible. The prediction proposed by Notter and Sleicher [9] is seen to be about 10% to 15% higher than the present result. This is also due to the different model used. The results of Na and Habib [14] seem to be in good agreement with those of the present study in a range of Peclet numbers, but their model is practical only when $Re > 10^4$ and $Pr > 0.02$.

Figure 9 depicts the fully developed Nusselt number vs the Reynolds number with the Prandtl number as a parameter. The dashed lines indicate the variation of Nusselt number with Peclet number as a parameter. It is observed that the solid lines coincide with each other as Reynolds number approaches a small value, say $Re = 3000$, and the curves with larger Prandtl numbers have larger Nusselt numbers before they coincide with the other curves. The reason is that for a given Reynolds number, eddy diffusivity increases as Prandtl number is increased as can be seen from equation (10), and thus a larger Nusselt number will be observed. It is noteworthy that for a small Reynolds number say $Re = 4000$, ε^+ is so small that eddy diffusivity is negligible compared to the molecular diffusivity for all Prandtl numbers in the range $0.001 \leqslant Pr \leqslant 0.02$ and therefore, the curves coincide in the low Reynolds number range. When the Reynolds number is less than 2300, the flow field should be laminar and the resulting Nusselt number is 4.364 for all Prandtl numbers. At a Peclet number below 450, say $Pe = 200$, increasing the Prandtl number should decrease the Reynolds number (since $Re = Pe/Pr$), and make the

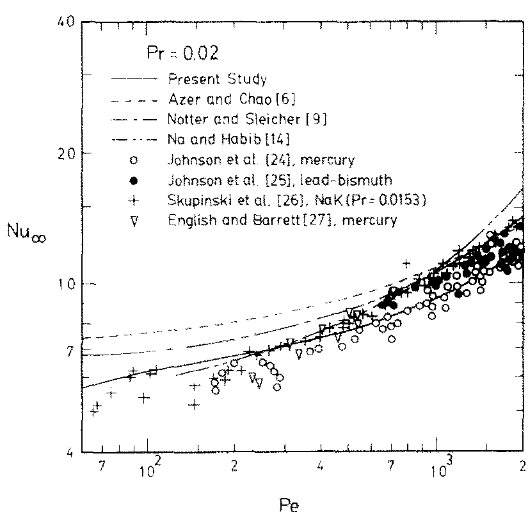

FIG. 8. Comparisons of existing experimental data and theoretical predictions with present study in thermally fully developed region.

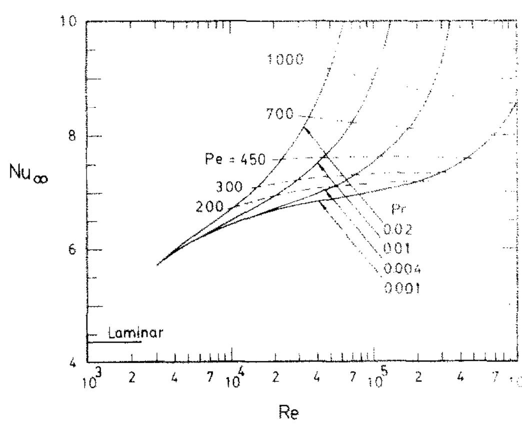

FIG. 9. Fully developed number vs Reynolds number with parameters Pr and Pe.

decrease in ε^+ more pronounced than the increase in Pr/Pr_t. This should cause a decrease in eddy diffusivity and thus a decrease in the Nusselt number. Therefore, the curve with $Pe = 200$ as shown in this figure has a positive slope. It is noted that the influence of laminar sublayer and buffer zone is significant at $Re < 10\,000$. At high Peclet number, say $Pe = 1000$, in both the cases of $Pr = 0.001$ and 0.02, the influence of boundary sublayers is insignificant and the eddy viscosity ε^+ is rather large. It means that the dimensionless total eddy diffusivity ε is much larger than unity and eddy diffusivity should increase as Prandtl number increases. Therefore, the curve with $Pe = 1000$ has a negative slope.

On the basis of the above discussion, there should exist a critical Peclet number near 450 whose slope is approximately zero. If the same results are depicted in Nusselt number vs Peclet number correlation with Prandtl number as a parameter as shown in Fig. 10, a crossover near $Pe = 450$ (but not exactly) would be observed. The Nusselt number is larger for a larger Prandtl number when $Pe > 450$ and the difference diverges as Peclet number approaches infinity. These findings are consistent with those reported by the existing predictions [6, 9]. The dashed lines indicate the variation of Nusselt number with Reynolds number as a parameter. It is seen that at low Reynolds number, say $Re = 10\,000$, increasing the Prandtl number increases the Nusselt number slightly for $Pe > 100$. Below $Pe = 100$, the fully developed Nusselt numbers seem to depend on Reynolds number only and the following interpolation formula

$$Nu_\infty = 3.01 Re^{0.0833} \qquad (17)$$

fits the calculated data well.

The series converges slowly at small Peclet numbers. In the present investigation, 20 terms are considered in the computations of the series expansions.

CONCLUSIONS

(1) The dimensionless temperature profile is lower and more uniform at the entrance, then increases along the longitudinal direction and finally develops into the fully developed profile θ_{fd} which results in a fully developed Nusselt number.

(2) For a given Prandtl number, the flow with larger Peclet number yields larger Nusselt number throughout $0 \leqslant x \leqslant \infty$. This is different from the case of laminar heat transfer.

(3) In thermally fully developed region, for a Peclet number below 450, increasing the Prandtl number should decrease the Nusselt number. The Nusselt number depends only on the Reynolds number at $Pe < 100$. The damping effect of the wall on Nusselt number is significant and the influence of boundary sublayers is not negligible at low Reynolds number.

(4) It is possible to have a fully developed Nusselt number less than 7 at low Peclet numbers.

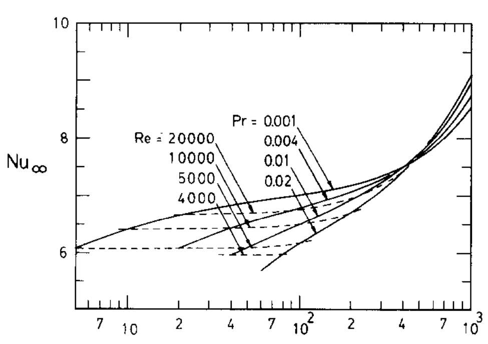

FIG. 10. Fully developed Nusselt number vs Peclet number with parameters Pr and Re.

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CONVECTION THERMIQUE D'UN METAL LIQUIDE EN ECOULEMENT TURBULENT DANS UN TUBE AVEC FLUX UNIFORME EN PAROI

Résumé—On présente l'étude du problème de l'établissement thermique pour un métal liquide en écoulement turbulent dans un tube rond. L'effet de la conduction axiale est prise en compte dans les directions en amont et en aval.

La solution exacte est obtenue en utilisant la méthode de séparation des variables pour $5 < Pe < 1000$ et $0{,}001 < Pr < 0{,}02$ avec $Re > 4000$. Les nombres de Prandtl couvrent le domaine complet des métaux liquides.

Un accord excellent est observé entre les nombres de Nusselt calculés et les données expérimentales à la fois pour la région d'entrée et pour la zone établie. La solution indique aussi que le nombre de Nusselt en zone pleinement établie dépend seulement du nombre de Reynolds aux faibles nombres de Péclet. La formule:

$$Nu_\infty = 3{,}01\ Re^{0{,}0833}$$

représente bien les calculs pour $Pe < 100$.

WÄRMEÜBERGANG VON FLÜSSIGEM METALL BEI TURBULENTER ROHRSTRÖMUNG UND GLEICHFÖRMIGER WÄRMESTROMDICHTE

Zusammenfassung—Diese Arbeit beschreibt eine Untersuchung zum Problem des Wärmeübergangs im thermischen Einlaufgebiet einer turbulenten Strömung von flüssigem Metall in einem Kreisrohr. Dabei wird der Einfluß der axialen Wärmeleitung entgegen der Strömung und in Strömungsrichtung berücksichtigt. Mit der Methode der Trennung der Variablen wird eine exakte Lösung für die Parameter Peclet-Zahl ($5 < Pe < 1000$) und Prandtl-Zahl ($0{,}001 \leqslant Pr \leqslant 0{,}02$) bei $Re > 4000$ erhalten. Die Prandtl-Zahlen decken das ganze Gebiet der flüssigen Metalle ab. Hervorragende Übereinstimmung erhält man bei einem Vergleich der errechneten Nußelt-Zahlen mit den verfügbaren experimentellen Daten sowohl im Einlaufgebiet als auch für die Gebiete der voll ausgebildeten Strömung. Ferner zeigt die Lösung, daß die Nußelt-Zahl im voll ausgebildetem Gebiet bei niedrigen Peclet-Zahlen nur von der Reynolds-Zahl abhängt. Die Interpolationsformel $Nu_\infty = 3{,}01 \cdot Re^{0{,}0833}$ gibt die berechneten Daten für $Pe < 100$ gut wieder.

ТЕПЛОПЕРЕНОС В ЖИДКИХ МЕТАЛЛАХ ПРИ ИХ ТУРБУЛЕНТНОМ ТЕЧЕНИИ В ТРУБАХ С ОДНОРОДНЫМ ПОТОКОМ ТЕПЛА НА СТЕНКЕ

Аннотация—Исследована задача о тепловом начальном участке для турбулентного потока жидкого металла в круглой трубе. При этом учитывается эффект осевой проводимости как в направлении, противоположном течению, так и вдоль него. Точное решение получено методом разделения переменных для параметров $5 < Pe < 1000$ и $0{,}001 \leq Pr \leq 0{,}02$ при $Re > 4000$. Значения числа Прандтля охватывают весь диапазон жидких металлов.

Показано хорошее соответствие полученных значений числа Нуссельта с имеющимися экспериментальными данными как на входе, так и в области полностью развитого течения. Решение также показывает, что число Нуссельта для полностью развитого участка зависит только от числа Рейнольдса при малых значениях числа Пекле. Найдено, что интерполяционное уравнение

$$Nu_\infty = 3{,}01\ Re^{0{,}0833}$$

хорошо согласуется с расчетными данными при $Pe < 100$.